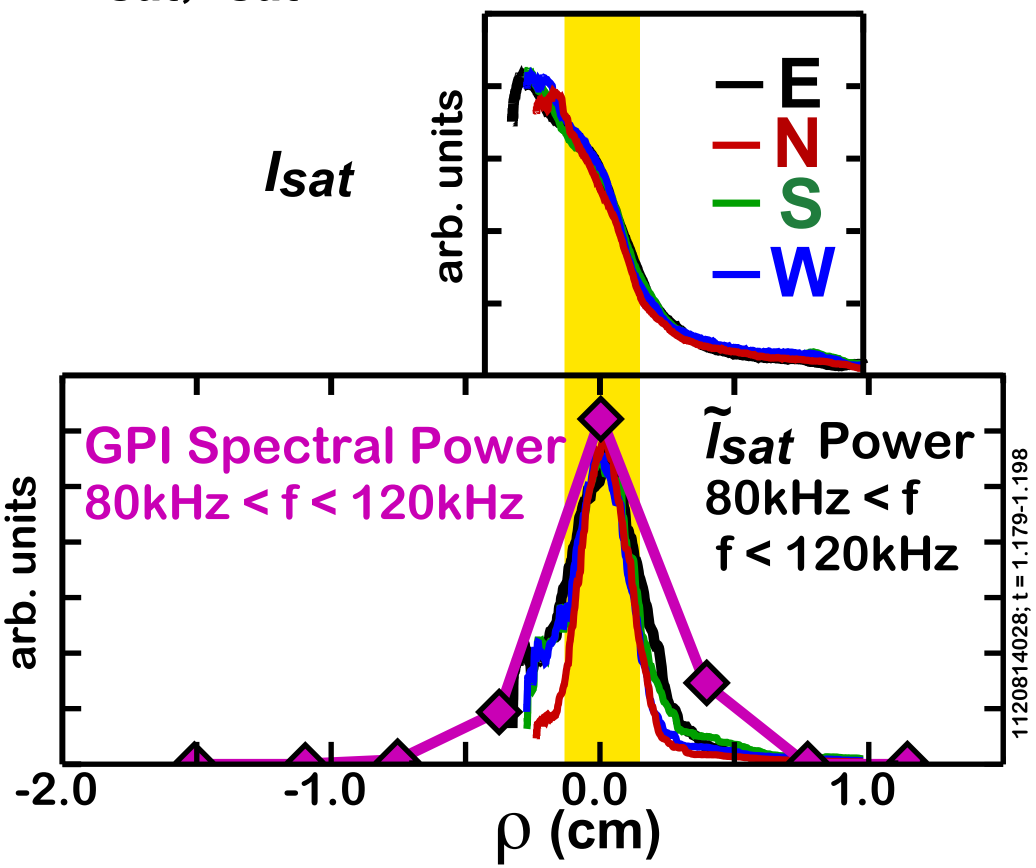
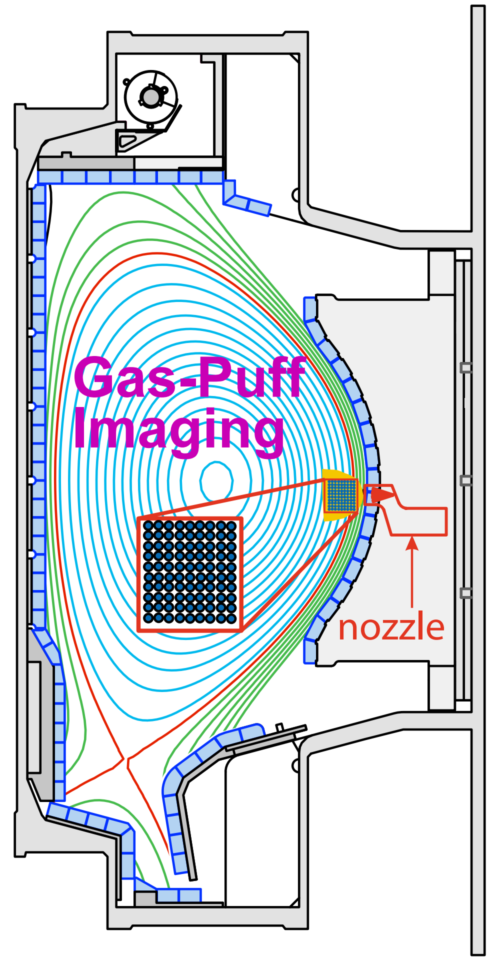
Physics and Theory of the Quasi-Coherent Mode

Narrow QCM layer width from ion saturation current fluctuations is consistent with Gas-Puff Imaging (GPI)

Alcator C-Mod

I_{sat}, $\tilde{I}_{sat}$ and *GPI* Fluctuation Profiles





- I_{sat} and $\tilde{I}_{sat}$ power profiles align, despite being recorded at different times by different probes
- Conclusion: QCM is ***not*** being attenuated by probe
- Narrow QCM layer is consistent with Gas-Puff Imaging (allowing shift)
- **Radial width of Quasi-Coherent Mode layer is ~ 3 mm FWHM**

Snapshot of QCM reveals large amplitude, ~in-phase, density, electron temperature and potential fluctuations

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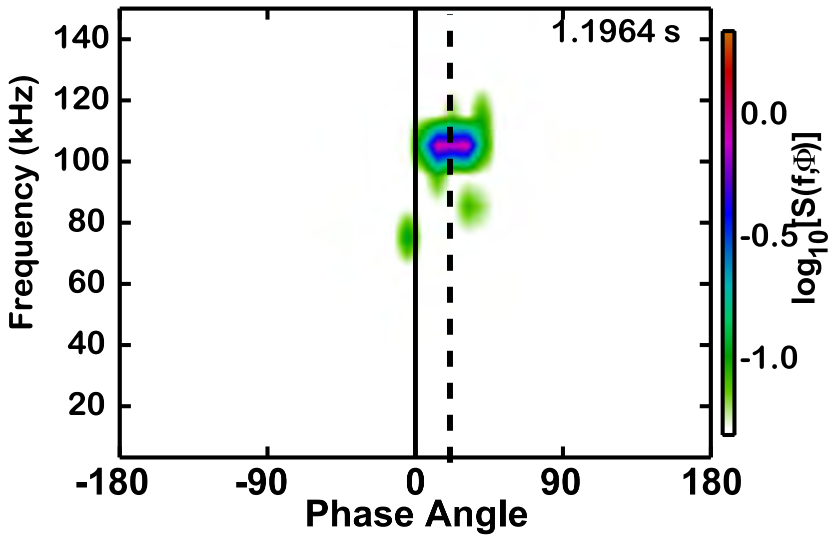
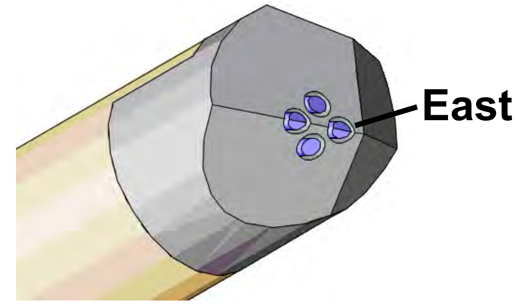
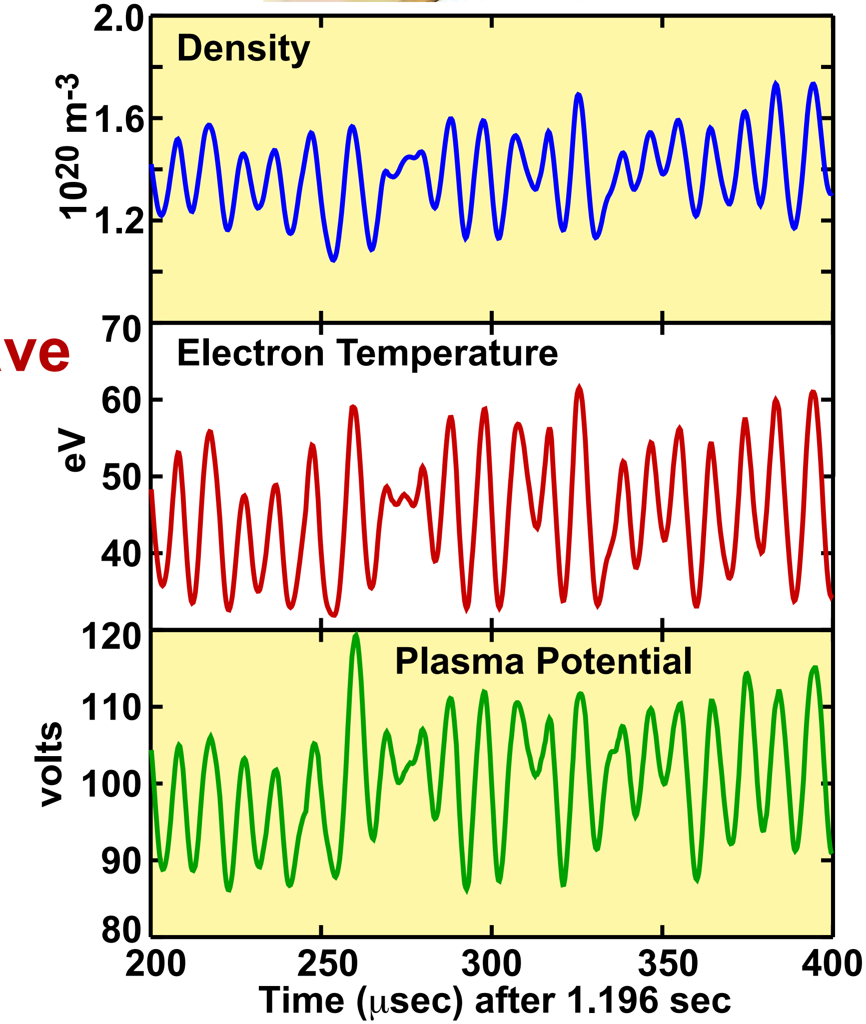
Cross Power Spectrum: Density and Potential



Potential lags Density with a phase angle of ~ 16 degrees

=> Drift wave

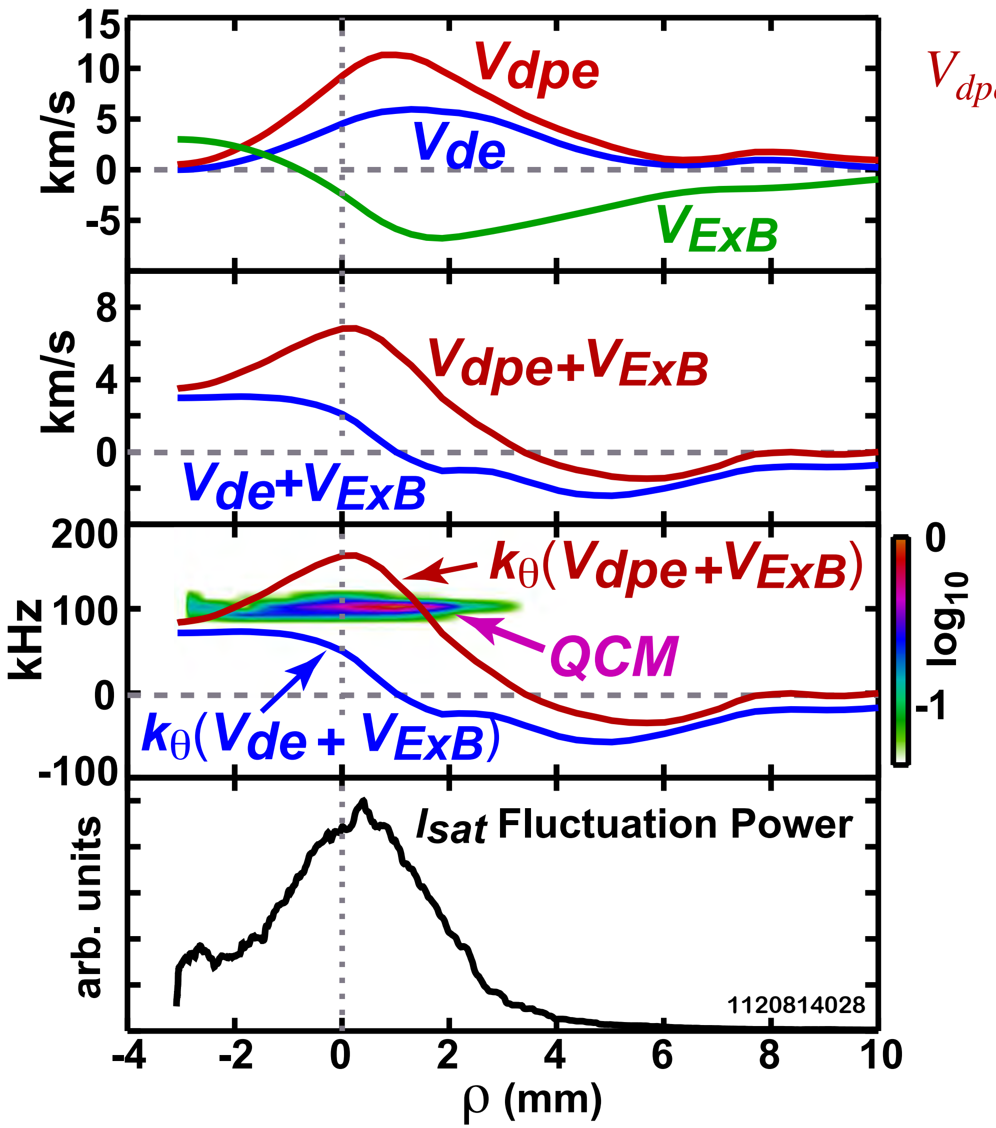




Quasi-coherent mode propagates at electron diamagnetic drift velocity in the plasma frame

Alcator C-Mod

Velocities computed from East electrode profiles



$$V_{dpe} = \frac{\nabla_r n T_e \times \underline{b}}{nB} \quad V_{de} = \frac{T_e \nabla_r n \times \underline{b}}{nB} \quad V_{ExB} = \frac{\underline{b} \times \nabla_r \Phi}{B}$$

- V_{dpe}, V_{de} are in opposite directions to V_{ExB} in mode layer

- V_{dpe}, V_{de} are stronger than V_{ExB} in mode layer

- QCM propagates in e^- dia. direction *in the plasma frame*

QCM frequency is quantitatively consistent with k_θ ~ 1.5 rad/cm mode propagating with velocity between V_{dpe} and V_{de} in the plasma frame.

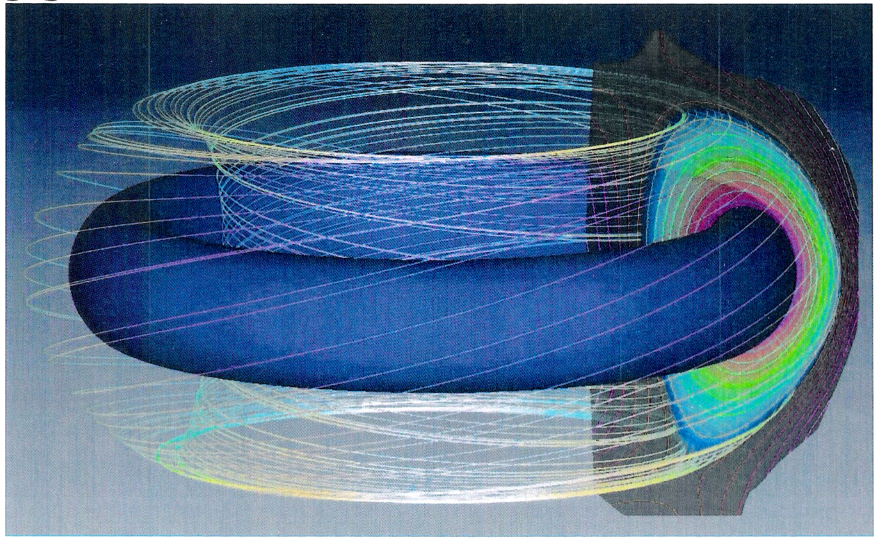
Toroidal magnetic field lines

Axisymmetric D-shaped tokamak with magnetic X-point(s) on separatrix.

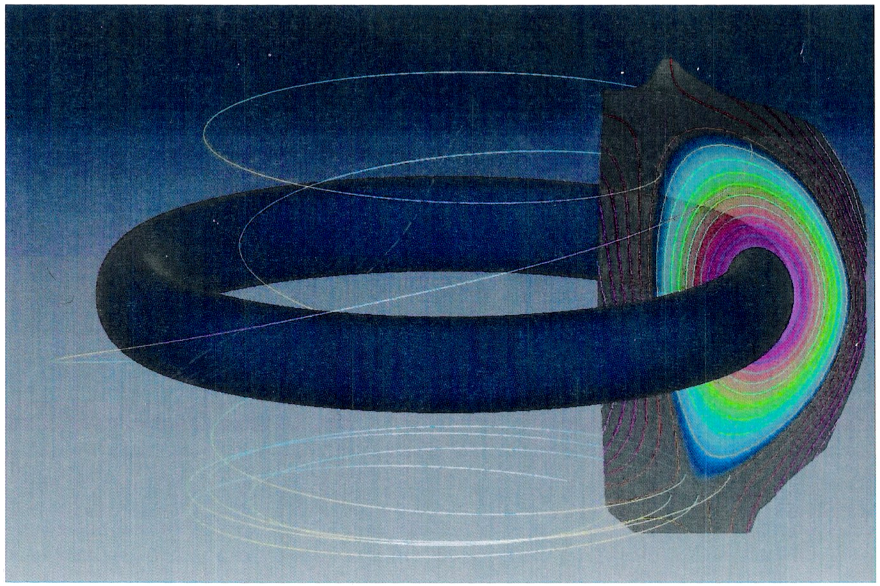
Field line winding number ι-bar(ψ) varies continuously from q≈1 (ι-bar=1/q) to plasma edge (0 on X-point separatrix).

Locally, field lines do most toroidal winding on top/bottom and inboard side of flux surface

Outboard side: from q≈1 to beyond separatrix, there is a vertical range centered on the midplane where the field lines have a fairly similar, finite pitch – B_θ/B_φ≈constant for C-Mod, DIII-D where a/R≈1/3.

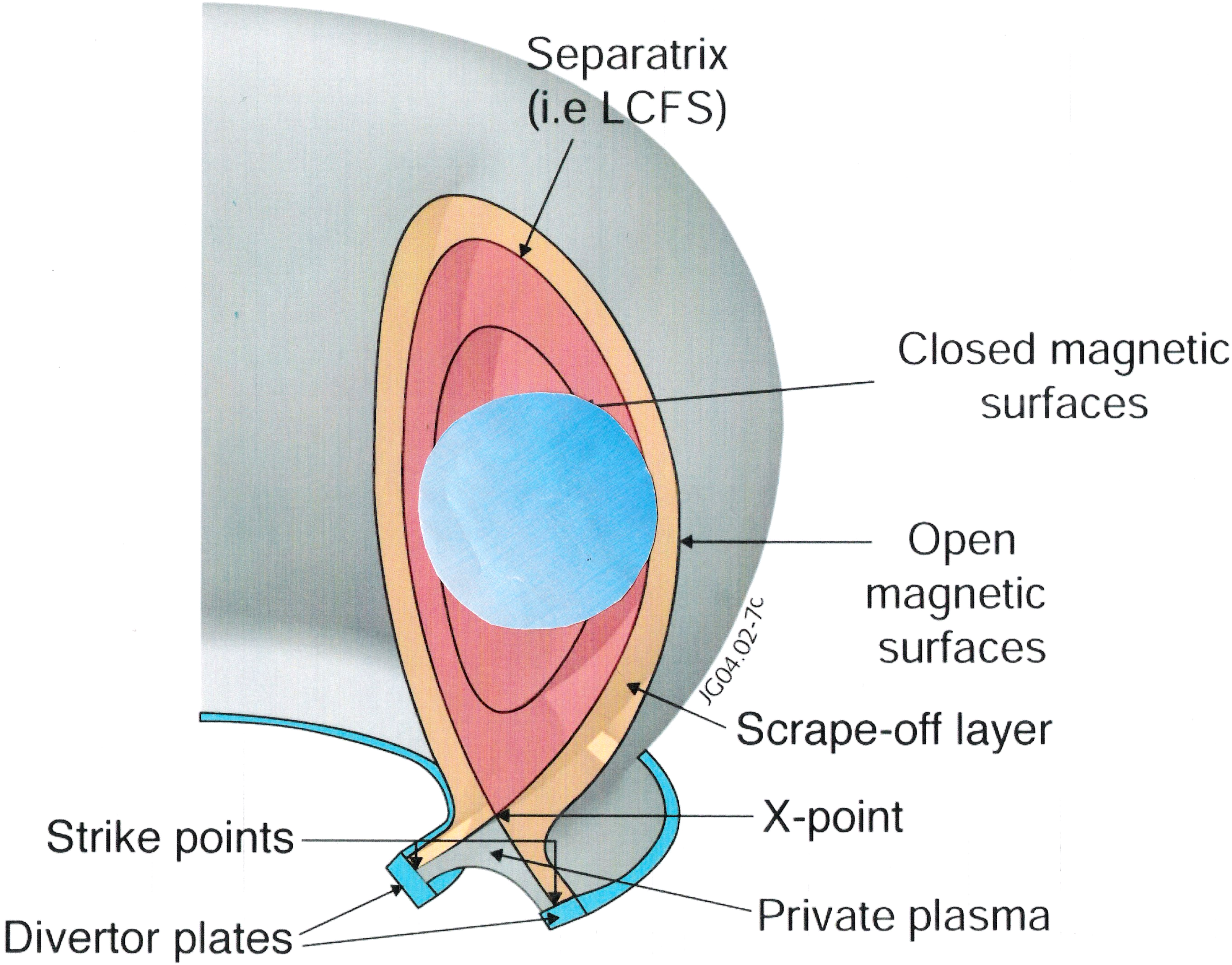


Just inside separatrix (DIII-D, lower X-pt)



Just outside separatrix

L. Sugiyama et al, Phys Plasmas 2010


Separatrix
(i.e LCFS)
Closed magnetic
surfaces
Open
magnetic
surfaces
JG04.02-7c
Scrape-off layer
X-point
Strike points
Private plasma
Divertor plates

for $-\theta_0 \leq \theta \leq \theta_0$

$$\mathbf{B} \simeq \frac{B_0 \mathbf{e}_\varphi + B_p(r_s)\mathbf{e}_\theta}{1+(r/R_0)\cos\theta}$$

$$\frac{B_p(r_s)}{B_0} \simeq const. \qquad (\text{for } -\theta_0 \leq \theta \leq \theta_0)$$

$$\hat{n} \simeq \tilde{n}(r-r_s,\theta)\exp\left\{-i\omega t + in^0\left[\varphi - q(r)\theta\right] + in^0\left[q(r)-q(r_s)\right]F_c(\theta-\theta_0)\right\}$$

$F_c(\theta) =$ correcting function acting at $\theta \simeq \pm\theta_0$

so that

$$\tilde{n}(r - r_s, \theta \geq \theta_0, \theta \leq -\theta_0) = 0$$

Radially Ballooning Modes

$$\tilde{n} \simeq \tilde{n}\left(\frac{r-r_s}{\delta_r}\right)\cos\left(\frac{\theta}{\theta_0}\frac{\Pi}{2}\right)F_c\left(r-r_s,\theta-\theta_0\right)$$

for

$$-\theta_0 < \theta < \theta_0$$

$$\frac{\partial^2}{\partial \ell^2} \simeq \left(\frac{B_\theta}{B r_s}\right)^2 \frac{\partial^2}{\partial \theta^2} \simeq -k_\parallel^2$$

$$k_\parallel^2 = \frac{B_\theta^2}{r_s^2 B^2}\left(\frac{1}{\theta_0}\frac{\Pi}{2}\right)^2$$

$$\left|\frac{\partial^2}{\partial r^2}\right| \gg \frac{m^{0\,2}}{r_s^2}$$

$$\gamma_G^2 \simeq \frac{V_{thi}^2\left(1+T_i/T_e\right)}{R_c}\left|\frac{1}{p}\frac{dp}{dr}\right|$$

$$-\frac{1}{r_p} \equiv \frac{1}{p}\frac{dp}{dr}$$

$$\left|\omega_{*e}^{p}\omega_{di}\right| \ll \gamma_G^2$$

$$\left(\frac{m^0}{r}\right)\rho_s\frac{V_s}{r_{pe}}\frac{\rho_i}{2}\frac{V_{thi}}{r_{pi}} \ll \gamma_G^2$$

Roughly

$$\left(\frac{m^0}{r}\right)^2 \rho_i^2 \ll \frac{r_p}{R_c}$$

Assume

$$\omega(\omega - \omega_{di})\frac{\partial^2}{\partial r^2} \sim \frac{m^{0\,2}}{r_s^2}\,\gamma_G^2$$

$$\delta_G^2 \sim \rho_i^2 \frac{R_c}{r_p}$$

$$-i\omega\hat{n}_e + \hat{V}_{Ex}\frac{\partial n}{dr} + n\frac{\partial}{\partial \ell}\hat{u}_{e\|} \simeq 0$$

$$\frac{3}{2}\left(-i\omega\hat{T}_e + \hat{V}_{Ex}\frac{dT_e}{dr}\right) + T_e\frac{\partial}{\partial \ell}\hat{u}_{e\|} \simeq 0$$

$$-i\left(\omega - k\frac{g_i}{\Omega_{ci}}\right)\hat{n}_i + \hat{V}_{Ex}\frac{dn}{dr} + n\nabla\cdot\left(\hat{\mathbf{V}}_{Pi} + \hat{\mathbf{V}}_{FLR}\right) \simeq 0$$

where

$$g_i \simeq \frac{V_{thi}^2}{R_e} \qquad x \equiv r - r_0$$

$$\hat{V}_{Ex} = -\frac{1}{r_s}\frac{\partial}{\partial\theta}\hat{\mathbf{\Phi}}\frac{c}{B}$$

$$\mathbf{\Omega}_{ci} = \frac{eB}{m_i c}$$

$$\frac{\partial}{\partial\ell} = \frac{\mathbf{B}}{B}\cdot\nabla$$

$$\hat{\mathbf{V}}_{Pi} = \text{polarization drift} \qquad \frac{c}{\mathbf{\Omega}_{ci}B}\frac{d\hat{\mathbf{E}}_\perp}{dt}$$

$$\hat{\mathbf{V}}_{FLR} \simeq \frac{\omega_{di}}{\omega} \hat{\mathbf{V}}_{Pi} \quad \text{Finite Larmor Radius Drift}$$

Plane Model

$$\eta_e \equiv \frac{d \ln T_e}{dx} \Big/ \frac{d \ln n}{dx}$$

$$\omega_{*e} = -k_y \frac{cT_e}{eBn} \frac{dn}{dx}$$

$$\omega_{di} = k_y \frac{c}{eBn} \frac{dp_i}{dx}$$

Then

$$\hat{n}_i \simeq -k\frac{c}{B}\hat{\Phi}\frac{\partial n}{\partial x}\bigg)_0 \frac{1}{\omega}\left(1+k\frac{g_i}{\omega\Omega_{ci}}\right)+\frac{n}{i\omega}\nabla\cdot\left(\hat{\mathbf{V}}_{Pi}+\hat{\mathbf{V}}_{FLR}\right)$$

$$\hat{n}_e \simeq -\frac{k}{\omega}\frac{c}{B}\Phi\frac{\partial n}{\partial x}+\frac{n}{i\omega}\frac{\partial}{\partial \ell}\hat{u}_{e\|} = \frac{\omega_{*e}}{\omega}\frac{e\hat{\Phi}}{T_e}n+\frac{n}{i\omega}\frac{\partial}{\partial \ell}\hat{u}_{e\|}$$

$$\frac{\partial}{\partial \ell}\hat{u}_{e\|} \simeq \nabla\cdot\left(\hat{\mathbf{V}}_{Pi}+\hat{\mathbf{V}}_{FLR}\right)-i\frac{k^2}{\omega}\frac{c}{B}\hat{\Phi}\frac{g_i}{\Omega_{ci}^n}\frac{\partial n}{\partial x}\bigg)_0$$

$$v_{ei}^{\|}nm_e\frac{\partial}{\partial \ell}\hat{u}_{e\|} \simeq -\frac{\partial^2}{\partial \ell^2}\left(\hat{n}_eT_e+n\hat{T}_e-e\hat{\Phi}n\right)$$

Plane Model

$$\omega \simeq \omega_{TT} \equiv \omega_{*e}\left[1 + (1 + \alpha_T)\eta_T\right]$$

$$\omega \simeq \omega_{TT}^{0} + \delta\omega$$

$$\delta\omega = i\gamma + \delta\omega_R$$

$x = r - r_0$

$$\omega_{TT} \simeq \omega_{TT}^{0}\left(1 - \frac{x^2}{\Delta_x^2}\right)$$

$$\left(\frac{\delta\omega}{\omega_{TT}\epsilon_\nu} + \frac{x^2}{\Delta_x^2 \epsilon_\nu}\right)\tilde{\phi}(x) \simeq i\left[1 - \delta_G^2 \frac{d^2}{dx^2}\right]\tilde{\phi}(x)$$

$$\tilde{\phi}(x) \propto \exp\left(-\sigma \frac{x^2}{2}\right)$$

$$\sigma = \sigma_R + i\sigma_I$$

$$\gamma \simeq \epsilon_\nu \omega_{TT}\left[1 + \frac{\delta_G}{(2\epsilon_\nu)^{1/2}\Delta_x}\right]$$

$$\epsilon_\nu \omega_{TT} \equiv k_y^2 \rho_s^2 \frac{\nu_{ei}'' m_e}{k_\parallel^2 T_e} \gamma_G^2$$

$$\gamma_G^2 \equiv \frac{g}{r_T}$$

$$\delta_G^2 \equiv \frac{\omega_{TT}(\omega_{TT} - \omega_{di})}{k_y^2 \gamma_G^2}$$

$$k_y^2 \simeq \left(\frac{m^0}{r_0}\right)^2$$

$$\delta_R \simeq \frac{1}{|2\epsilon_\nu|^{1/2} \Delta_x \delta_G}$$

$$\delta_r \cong |2\epsilon_\nu|^{1/4} (\Delta_x \delta_G)^{1/2}$$

$$\gamma \simeq \left(\frac{m^0}{r_0}\rho_s\right)^2 \frac{\nu_{ei}^{\parallel} m_e}{k_\parallel^2 T_e} \gamma_G^2 \left[1 + \frac{\delta_G}{\Delta_x |2\epsilon_\nu|^{1/2}}\right]$$

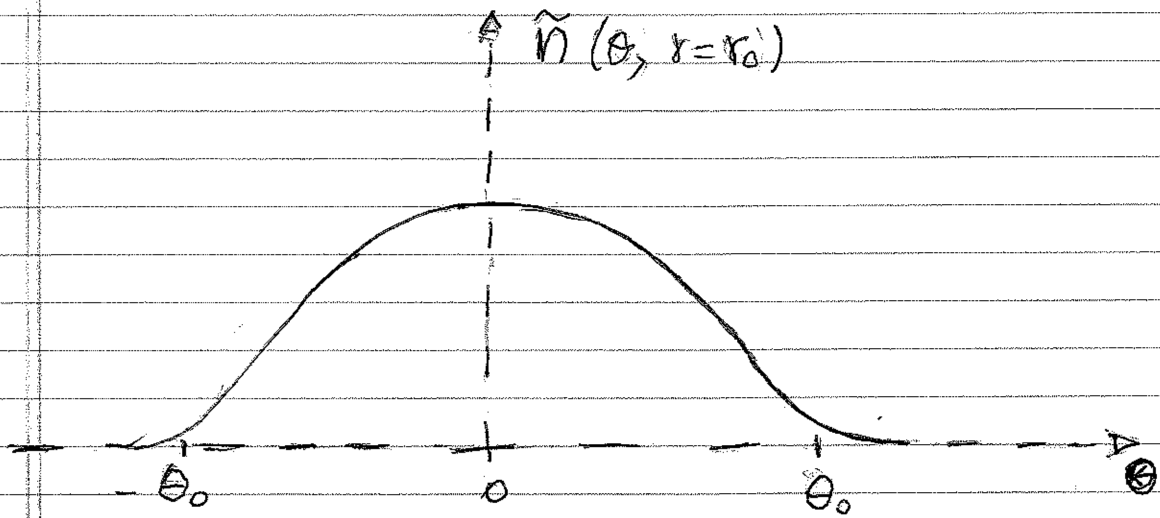
or

$$\gamma \simeq \left(\frac{m^0}{r_0}\rho_s\right)^2 \frac{\nu_{ei}^{\parallel} m_e}{k_\parallel^2 T_e} \gamma_G^2 + \left(\frac{\nu_{ei}^{\parallel} m_e}{2 k_\parallel^2 T_e}\right)^{1/2} \gamma_G |\omega_{*T}^0| \frac{\rho_s}{\Delta_x}$$

$$\times \left(1 - \frac{\omega_{di}}{\omega_{TT}}\right)^{1/2}$$

2-D Peaking Mode?

$$\hat{n} \simeq \tilde{n}\left(r - r_s, \theta\right)\exp\left\{-i\omega t + in^0\left[\varphi - q\left(r_s\right)\theta\right] - in^0\left[q\left(r\right) - q\left(r_s\right)\right]F\left(\theta\right)\right\}$$



For

$$-\theta_0 + \Delta\theta_0 < \theta \leq \theta_0 - \Delta\theta_0$$

$$\tilde{n} \propto \cos\left(\frac{\pi}{2}\frac{\theta}{\theta_0}\right)$$

Electron Thermal Energy Balance Equation

Simplest (cylindrical) form

$$\left(-i\omega + D_{\|}^{2}k_{\|}^{2}\right)\hat{T}_e + T_e'\left(\hat{V}_{Er} - ik_{\|}\frac{\hat{B}_r}{B}D_{\|}\right) + ik_{\|}T_e\hat{U}_{e\|}\frac{2}{3}\left(1+\alpha_T\right) \simeq 0$$

That gives

$$\hat{T}_e \simeq \frac{1}{\left(\omega + iD_{\|}k_{\|}^{2}\right)}\left[\left(\omega\hat{\xi}_r + k_{\|}\frac{\hat{B}_r}{B}D_{\|}\right)\left(-T_e'\right) + \frac{2}{3}k_{\|}\hat{U}_{e\|}T_e\left(1+\alpha_T\right)\right]$$

for $\hat{V}_{Er} = -i\omega\hat{\xi}_r$

Appendix

Now we give the approximate numerical estimates for a set of parameters that are involved in the theory of the Quasi-Coherent Mode discussed earlier. These estimates are based on the relevant experimental observations [1] made by the Alcator C-Mod machine.

- Frequency Range

$$f \sim 100\ \text{kHz}, \qquad \omega \sim 6.3 \times 10^5\ \ \text{rad} \cdot \text{sec}^{-1}.$$

- Major Radius of the Plasma Column

$$R_0 \simeq 68\ \text{cm}.$$

- Location of the Mode Center R_{mc}

$R_{mc} \simeq R_{_{LCFS}}$ — LCFS stands for the Last Closed Flux Surface.

- Mode Radial Width

$$\Delta r \simeq 3\ \text{mm}.$$

- Sign of E_r inside the mode layer

$$E_r = -\frac{\partial \phi}{\partial r} > 0.$$

Therefore $v_{_E}/v_{di} > 0$.

- Range of Poloidal Mode Phase Velocity $v_{ph} \equiv \omega/k_\theta$

$$v_{*e} < v_{ph} - v_E < v_{de}$$

where the electron temperature gradient is significant across the layer in which the QCM is excited.

- Density Fluctuation Level

$$\frac{\tilde{n}}{n} \sim 30\%.$$

- Electron Temperature Fluctuation Level

$$\frac{\tilde{T}_e}{T_e} \sim 45\%.$$

- Electric Potential Fluctuation Level

$$\frac{e\tilde{\phi}}{T_e} \sim 45\%.$$

- Electron Temperature at R_{LCFS}

$$T_e \simeq 50 \ \ \text{eV}.$$

- Electron Density at $R \simeq R_{LCFS}$

$$n_e \simeq 1.5 \times 10^{20} \ \text{m}^{-3}.$$

- Poloidal Wavenumber

$$k_\theta \sim 1.5 \ \text{rad/cm}.$$

- Thermal Velocities

$$v_{thi} \simeq 6.9 \times 10^6 \left[\frac{T_i}{50\,\mathrm{eV}}\right]^{\frac{1}{2}} \mathrm{cm} \cdot \mathrm{sec}^{-1} \qquad \text{deuteron}$$

$$v_{the} \simeq 3.0 \times 10^8 \left[\frac{T_e}{50\,\mathrm{eV}}\right]^{\frac{1}{2}} \mathrm{cm} \cdot \mathrm{sec}^{-1} \qquad \text{electron}$$

- Collisional Frequencies

deuterons

$$\nu_{ii} \simeq 1.73 \times 10^5 \left[\frac{n_i}{1.5 \times 10^{14}\,\mathrm{cm}^{-3}}\right] \left[\frac{\ln\Lambda}{12}\right] \left[\frac{50\,\mathrm{eV}}{T_i}\right]^{\frac{3}{2}} \mathrm{sec}^{-1}.$$

electron - deuteron

$$\nu_{ei} \simeq 1.48 \times 10^7 \left[\frac{n_i}{1.5 \times 10^{14}\,\mathrm{cm}^{-3}}\right] \left[\frac{\ln\Lambda}{12}\right] \left[\frac{50\,\mathrm{eV}}{T_i}\right]^{\frac{3}{2}} \mathrm{sec}^{-1}.$$

- Mean Free Paths

$$\lambda_{ii} \simeq 4.0 \times 10^1 \left[\frac{v_{thi}}{6.9 \times 10^6\,\mathrm{cm} \cdot \mathrm{sec}^{-1}}\right] \cdot \left[\frac{1.73 \times 10^5\,\mathrm{sec}^{-1}}{\nu_{ii}}\right] \mathrm{cm}.$$

$$\lambda_{ee} \simeq 2.0 \times 10^1 \left[\frac{v_{the}}{3.0 \times 10^8\,\mathrm{cm} \cdot \mathrm{sec}^{-1}}\right] \cdot \left[\frac{1.48 \times 10^7\,\mathrm{sec}^{-1}}{\nu_{ee}}\right] \mathrm{cm}.$$